



The Air Force Research Laboratory (AFRL)



**Wright-Patterson
Educational Outreach**

John C. Sparks

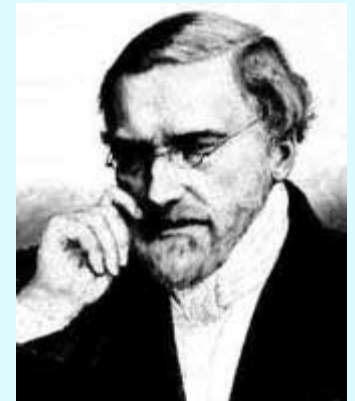
Basic Terminal Ballistics





French Mathematician: Jean-Victor Poncelet (1788-1867)

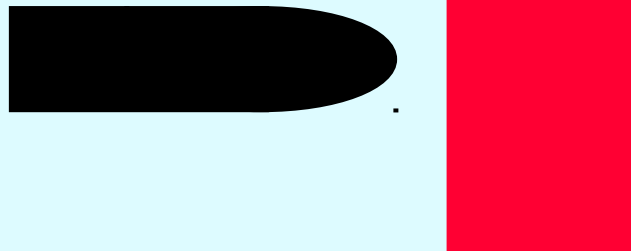
- ◆ French mathematician and engineer
- ◆ Served as a 'Lieutenant of Engineers' under Napoleon in War of 1812
- ◆ Abandoned as dead during Russian campaign
 - ➔ Captured and imprisoned by Russians at Saratov
 - Released by Russians in 1814
- ◆ Mathematical achievements
 - Father of modern projective geometry
 - Co-verifier of Feuerbach's 9-point circle theorem
 - Proposed Poncelet-Steiner Euclidian construction theorem—now proved
- ◆ Engineering achievements: founded the **Science of Terminal Ballistics**
 - Alternately called Penetration Mechanics today





Poncelet Differential Equation for Bullet Penetration

$$\frac{d}{dt}(mv) = -A_{cs} \cdot c_0 - A_{cs} \cdot c_1 \cdot v^2$$



In words: The instantaneous *time-rate-of-change* of bullet momentum equals the sum of two retarding forces, a general form drag which is proportional to the cross-sectional area of the penetrator and a dynamic drag term jointly proportional to the cross-sectional area of the penetrator times penetrator velocity squared (e.g. a kinetic-energy-like term). The two constants of proportionality are deemed primarily dependent on the material being penetrated.



The Poncelet Dilemma: How to Determine c_0 and c_1 from Data

$$\frac{d}{dt}(mv) = -A_{cs} \cdot c_0 - A_{cs} \cdot c_1 \cdot v^2$$

$x=0,$
 $t=0,$
 $v(0)=$
 v_s



$x=L,$
 $t=?,$
 $v(?)=0$

How can we use what is measurable to completely characterize the ballistic-penetration sequence?



Answer: by Changing the Independent Variable

$$\frac{d}{dt}(mv) = m \frac{d}{dt}(v) = m \cdot \frac{dv}{dx} \cdot \frac{dx}{dt} \Rightarrow$$

$$\frac{d}{dt}(mv) = m \cdot v \cdot \frac{dv}{dx}$$

In his development, Poncelet assumed that there was no significant mass loss during the bullet's travel inside the material being penetrated. This is not always true in today's world of liquefying penetrators and ablation-type phenomena. Another assumption is that cross-sectional areas remain constant, which does not hold true for expanding or mushrooming bullets. But then again, be kind to Poncelet; for he did this pioneering work circa 1850!



Poncelet's Transformed Differential Equation

$$mv \frac{dv}{dx} = - A_{cs} \cdot c_0 - A_{cs} \cdot c_1 \cdot v^2 \Rightarrow$$

$$v \frac{dv}{dx} = - B \cdot c_0 - B \cdot c_1 \cdot v^2 : B = \frac{A_{cs}}{m} \Rightarrow$$

$$v \frac{dv}{dx} = - B \cdot (c_0 + c_1 \cdot v^2)$$

$$B.C. \rightarrow v(0) = v_s, v(L) = 0$$



Poncelet Penetration Equation: Step 1 of Solution Process

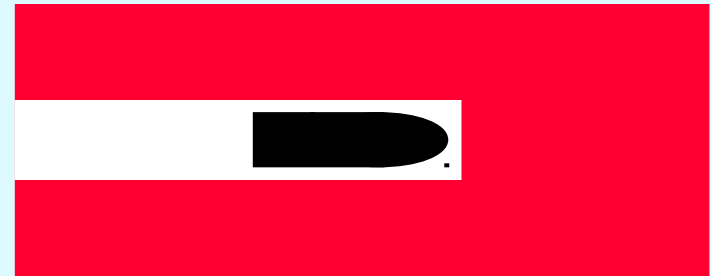
$$v \frac{dv}{dx} = -B \cdot (c_0 + c_1 v^2) \Rightarrow$$

$$\frac{v dv}{c_0 + c_1 v^2} = -B dx \Rightarrow$$

$$\int \frac{v dv}{c_0 + c_1 v^2} = \int (-B) dx + K \Rightarrow$$

$$\frac{1}{2c_1} \ln |c_1 v^2 + c_0| = -Bx + K \Rightarrow$$

$$c_1 v^2 + c_0 = K_1 e^{-2c_1 Bx}$$



Separation of
independent and
dependent variables
works quite nicely
here



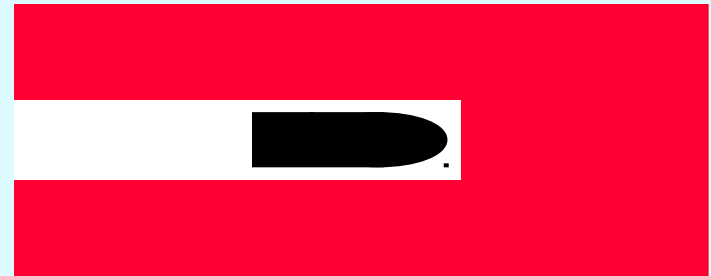
Poncelet Penetration Equation: Step 2 of Solution Process

$$v(0) = v_s \Rightarrow$$

$$K_1 = c_1 v_s^2 + c_0 \Rightarrow$$

$$c_1 v(x)^2 + c_0 = (c_0 + c_1 v_s^2) e^{-2c_1 Bx}$$

$$v(x) = \sqrt{\frac{(c_0 + c_1 v_s^2) e^{-2c_1 Bx} - c_0}{c_1}}$$



Apply the
boundary or
initial condition

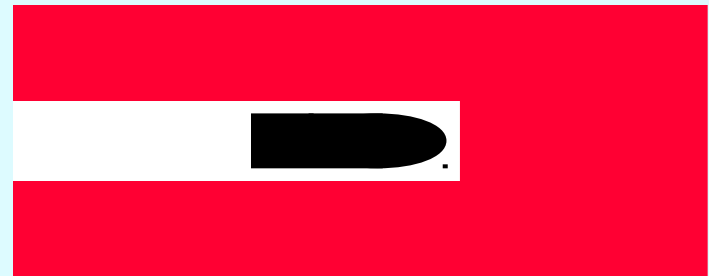


Immediate Result: An Equation for Maximum Penetration Depth

$$v(L) = \sqrt{\frac{(c_0 + c_1 v_s^2) e^{-2c_1 B L} - c_0}{c_1}} = 0 \Rightarrow$$

$$(c_0 + c_1 v_s^2) e^{-2c_1 B L} - c_0 = 0 \Rightarrow$$

$$L = \frac{1}{2c_1 B} \ln \left[\frac{c_0 + c_1 v_s^2}{c_0} \right]$$





Summary of Poncelet's *Hybrid* Ballistic Penetration Methodology

- ◆ Methodology grounded in classical Newtonian Physics:
 $F=MA$
 - ➔ Incorporates obvious parameters: striking velocity, mass, and cross-sectional area
 - ▢ Incorporates two obvious retarding forces: form (or geometric) drag and dynamic drag
 - ▢ Physical characteristics of system are assumed constant—no mass loss, shape change, liquefaction, ablation, etc.
- ◆ Methodology incorporates two unknown parameters (*hybrid*)
 - ▢ Assumed to be material and interface dependent—hence can be viewed as material properties
 - ▢ Properties must be determined via testing
- ◆ Newton's Law of Cooling is also a hybrid methodology due to the heat-transfer coefficient h in $\frac{dT}{dt} = -h(T - T_A)$